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A vector-like heavy quark in the littlest Higgs model

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Abstract: The littlest Higgs model contains a new vector-like heavy quark in the up sector. There are two interesting features of its existence. One is that it extends the 3×3 CKM matrix in the standard model to a 4×3 matrix and the other is that it allows Z-mediated flavor changing neutral currents at tree level in the up sector but not in the down sector. We examine a few of possible windows in which the Z-mediated flavor changing neutral currents in the littlest Higgs model can be tested.

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1. Introduction

A variety of experiments has revealed three generations for quarks in the standard model. Theoretically the cancellation of the gauge anomalies explains the equal number of weak doublets of quarks and leptons. This also account for three quark generations. The number of quark generations implies the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix for the quark mixing. Weak decays of the relevant quarks or deep inelastic neutrino scattering can determine all the values of the CKM matrix elements. Therefore, the number of quark generations, in principle, can be tested by measuring the CKM matrix. So far the experimental data have not precluded there being more than three generations.

A new theory for electroweak symmetry breaking was developed from deconstruction theory two years ago [1, 2]. This theory is named as "the littlest Higgs (LH) model" because it is the smallest extension of the SM to date that stabilizes the mass of the SM Higgs boson on the electroweak scale. The SM Higgs doublet belongs to a Goldstone multiplet in an SU(5)/ SO(5) nonlinear sigma model. Other elements in the multiplet become a Higgs triplet. The nonlinear transformation of the Goldstone bosons under collective global symmetries naturally ensures the absence of the SM Higgs mass term of the form $m^2|h|^2$ at a TeV energy scale. The Higgs mass squared parameter arises from the Coleman-Weinberg potential in the gauge sector as well as in the fermion sector at the electroweak scale [3].

In the LH model the hierarchy problem of the SM is naturally solved in a similar way as in supersymmetric extensions of the SM: The one-loop divergences from the SM particles of spins $J = 1, 1/2, 0$ on the Higgs mass parameter are cancelled by those from new massive particles of the same spins $J = 1, 1/2, 0$, respectively [4]. Among the new heavy particles the fermion with spin $J = 1/2$ is a vector-like heavy quark that is analyzed in this paper. The existence of the heavy quark introduces new effects in the weak currents. The CKM matrix is extended to 4×3 and flavor changing neutral currents occur at tree level.

This article is organized as follows. In section 2 we review the LH model. We describe the gauge sector as well as the fermion sector in detail. In section 3 we study the charged currents in the LH model and introduce the extended 4×3 CKM matrix. In section 4 we discuss the flavor changing nuetral currents in the LH model and the neutral mixing angles. We present a few of measurements which determine the mixing angles. In section 5 we draw a conclusion. We present the detailed derivation of the neutral mixing anlges from the up-type quark mass matrix and the Yukawa couplings in the appendix.

2. The littlest Higgs model

The littlest Higgs (LH) model begins with SU(5) global symmetry, with a locally gauged subgroup $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ [1]. The SU(5) global symmetry is spontaneously broken down to its subgroup SO(5) at the scale $f \sim 1\,\mathrm{TeV}$. The vacuum expectation value associated with the spontaneous symmetry breaking is proportional to the 5×5 symmetrical matrix

$$\Sigma_0 = \begin{pmatrix} & & \mathbb{1}_{2\times 2} \\ & 1 & \\ \mathbb{1}_{2\times 2} & & \end{pmatrix}. \tag{2.1}$$

The global symmetry breaking results in fourteen massless Goldstone bosons. Among them four massless Goldstone bosons are eaten by the gauge bosons so that the gauge group $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ is broken down to its diagonal subgroup $\mathrm{SU}(2) \times \mathrm{U}(1)$. The remaining ten Goldstone bosons can be parameterized by a non-linear field Σ as follows:

$$\Sigma = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f}\Sigma_0 \tag{2.2}$$

$$\Pi = \begin{pmatrix} & \frac{h^\dagger}{\sqrt{2}} & \phi^\dagger \\ \frac{h}{\sqrt{2}} & & \frac{h^*}{\sqrt{2}} \\ \phi & \frac{h^T}{\sqrt{2}} & \end{pmatrix}. \tag{2.3}$$

Here Π is the Goldstone bosons which fluctuate about this background in the broken directions. The field contents are grouped as a complex doublet h and a complex triplet ϕ with hypercharge $Y_h = 1/2$ and $Y_\phi = 1$ under the unbroken gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$:

$$h = \begin{pmatrix} h^+ & h^0 \end{pmatrix}, \qquad \phi = \begin{pmatrix} \phi^{++} & \frac{\phi^+}{\sqrt{2}} \\ \frac{\phi^+}{\sqrt{2}} & \phi^0 \end{pmatrix}. \tag{2.4}$$

The complex doublet becomes the SM Higgs doublet while the complex triplet is an addition to the SM particle contents. The triplet acquires a TeV scale mass at one-loop from the Coleman-Weinberg potential in the gauge sector as well as in the fermion sector.

The SM Higgs doublet acquires the mass squared parameter at two-loop as well as at one-loop from the Coleman-Weinberg potential. On the other hand, the SM Higgs quartic self-coupling arises after integrating over the massive triplet. The one-loop quadratic divergence on the SM Higgs mass parameter from the SM Higgs self coupling is naturally cancelled by that from the triplet coupling to the SM Higgs doublet.

2.1 Gauge sector

The LH model has a broken SU(2) × U(1) symmetry as well as the unbroken standard model $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry at the scale f. This is a key feature of the Little Higgs construction. Though the pseudo Goldstone multiplet vary from model to model all the Little Higgs models have extended gauge groups. The quadratic divergence of the SM gauge bosons in the Higgs mass is cancelled by that of a heavy copy of the standard model gauge boson. On the other hand, In the sypersymmetric extension of the SM model the divergence of the standard model gauge bosons in the Higgs mass is cancelled by the gaugino contribution.

The gauge group structure in the LH model is given as follows. The generators of the SU(2)'s are embedded into SU(5) as

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & \\ & \mathbf{0}_{3\times 3} \end{pmatrix}, \qquad Q_2^a = \begin{pmatrix} \mathbf{0}_{3\times 3} & \\ & \sigma^{a*}/2 \end{pmatrix} \tag{2.5}$$

while the generators of the $U(1)$'s are given by

$$Y_1 = \begin{pmatrix} -\frac{3}{10}\mathbb{1}_{2\times 2} & \\ & \frac{2}{10}\mathbb{1}_{3\times 3} \end{pmatrix}, \qquad Y_2 = \begin{pmatrix} -\frac{2}{10}\mathbb{1}_{3\times 3} & \\ & \frac{3}{10}\mathbb{1}_{2\times 2} \end{pmatrix}. \tag{2.6}$$

The generators of the electroweak symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are expressed by $Q^a = \frac{1}{\sqrt{2}}(Q_1^a + Q_2^a)$ and $Y = Y_1 + Y_2$. The kinetic term for the pseudo Goldstone bosons can be written as

$$\mathcal{L}_\Sigma = \frac{1}{2}\frac{f^2}{4}\,\mathrm{Tr}\,|\mathcal{D}_\mu \Sigma|^2\,, \tag{2.7}$$

where the covariant derivative is given by

$$\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma + i\sum_{j=1}^{2}\left[g_j(W_j\Sigma + \Sigma W_j^T) + g_j'(B_j\Sigma + \Sigma B_j^T)\right], \tag{2.8}$$

with the gauge bosons are defined by

$$W_j = \sum_{a=1}^{3} W_{\mu j}^a Q_j^a, \quad B_j = B_{\mu j} Y_j\,. \tag{2.9}$$

The spontaneous gauge symmetry breaking gives rise to mass terms of order f for the broken gauge bosons

$$\begin{aligned}\mathcal{L}_{\Sigma,\mathrm{mass}} = &\frac{1}{2}\frac{f^2}{4}\left[g_1^2 W_{1\mu}^a W_1^{a\mu} + g_2^2 W_{2\mu}^a W_2^{a\mu} - 2g_1 g_2 W_{1\mu}^a W_2^{a\mu}\right] + \\ &+ \frac{1}{2}\frac{f^2}{4}\frac{1}{5}\left[g_1'^2 B_{1\mu}B_1^\mu + g_2'^2 B_{2\mu}B_2^\mu - 2g_1' g_2' B_{1\mu}B^{2\mu}\right].\end{aligned} \tag{2.10}$$

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The W, W_H, B and B_H fields in mass eigenstates are defined as

$$\begin{aligned} W &= \sin\theta W_1 + \cos\theta W_2\,, \qquad W_H = -\cos\theta W_1 + \sin\theta W_2 \\ B &= \sin\theta' B_1 + \cos\theta' B_2\,, \qquad B_H = -\cos\theta' B_1 + \sin\theta' B_2\,, \end{aligned} \tag{2.11}$$

where the mixing angles are given by

$$\begin{aligned} \sin\theta &= \frac{g_2}{\sqrt{g_1^2+g_2^2}}\,, \qquad \cos\theta = \frac{g_1}{\sqrt{g_1^2+g_2^2}} \\ \sin\theta' &= \frac{g_2'}{\sqrt{g_1'^2+g_2'^2}}\,, \qquad \cos\theta' = \frac{g_1'}{\sqrt{g_1'^2+g_2'^2}}\,. \end{aligned} \tag{2.12}$$

At the energy scale f, the SM guage bosons W and B are massless and the masses of the heavy gauge bosons W_H and B_H are then given by

$$M_{W_H} = \frac{f}{2}\sqrt{g_1^2+g_2^2}\,, \qquad M_{B_H} = \frac{f}{2\sqrt{5}}\sqrt{g_1'^2+g_2'^2}\,. \tag{2.13}$$

2.2 Fermion sector

In the standard model the Yukawa couplings for all fermions besides the top quark are small, and it is not necessary to protect the Higgs mass from their one-loop quadratic divergences. The top Yukawa coupling is the strongest one-loop quadratic divergence in the standard model, so its negative contribution to one-loop quadratic divergence dominates all the positive contributions from the gauge sector as well as from the Higgs sector in the standard model.

In the LH model we incorporate dominance of the top quark one-loop divergence in the standard model with the introduction of the Higgs triplet and new heavy gauge bosons in order to trigger the electroweak symmetry breaking. The Higgs doublet is a Goldstone boson at the scale f so that there is no mass term for the Higgs doublet at the tree level. The Higgs potential arises from the Coleman-Weinberg potential. Additional heavy gauge bosons and the Higgs triplet give the Higgs doublet a logarithmically enhanced positive mass squared. In order to cancel out these positive contributions and get a negative Higgs mass squared we introduce an additional quark in vector-like representation of the standard model gauge group [6],

$$\tilde{t}(3,1)_{+2/3} + \tilde{t}^c(\bar{3},1)_{-2/3}\,. \tag{2.14}$$

Note that it is a singlet under the $\mathrm{SU}(2)_L$ gauge group to evade the gauge anomalies.

The LH model contains Yukawa couplings between the fermions and the scalars. A large top Yukawa coupling arises because the Weyl fermions $\tilde{t}, \tilde{t}^c$ mix mostly with the usual third-generation weak doublet $q_3 = (b_3, t_3)$ and weak singlet $u_3'^c$. The interaction between the Σ field and quarks in the up sector can be taken as

$$\mathcal{L}_{U,\mathrm{mass}} = \frac{1}{2}\lambda^U_{ab} f \epsilon_{ijk}\Psi_{ai}\Sigma_{jx}\Sigma_{ky}u_b'^c + \lambda_0 f\tilde{t}\tilde{t}^c + \mathrm{h.c.} \tag{2.15}$$

$$\Psi_{1i} = \begin{pmatrix} d_1 \\ u_1 \\ 0 \end{pmatrix}, \qquad \Psi_{2i} = \begin{pmatrix} s_2 \\ c_2 \\ 0 \end{pmatrix}, \qquad \Psi_{3i} = \begin{pmatrix} b_3 \\ t_3 \\ \tilde{t} \end{pmatrix}, \tag{2.16}$$

where λ_{ab}^U are the ordinary Yakawa couplings in the up sector and λ_0 is a new Yukawa coupling. This is a slight but natural generalization of the scalar couplings to the quark in the up sector. Note that mixing between different generations occurs. Expanding the Σ field generates the Higgs interactions with quarks. After the Higgs doublet gets a vev v the up-type quark mass matrix $\mathcal{M}_U$ becomes

$$\mathcal{M}_U = \begin{pmatrix} i\lambda_{11}v & i\lambda_{12}v & i\lambda_{13}v & 0 \\ i\lambda_{21}v & i\lambda_{22}v & i\lambda_{23}v & 0 \\ i\lambda_{31}v & i\lambda_{32}v & i\lambda_{33}v & 0 \\ 0 & 0 & \lambda_{33}f & \lambda_0 f \end{pmatrix}. \tag{2.17}$$

Here we ignore the triplet vev v' because its value is much smaller than the doublet vev, v [2, 7]. Note that the element (3,4) of the matrix $\mathcal{M}_U$ is zero because there is no mixing between t_3 and $\tilde{t}^c$ in the lagrangian. The weak eigenstate U_L and mass eigenstate U_L^m are related by $U_L^m = T_L^U U_L$ with $U_L^m = (u_L, c_L, t_L, T_L)^T$ and $U_L = (u_1, c_2, t_3, \tilde{t})^T$. The Higgs interactions with the down-type quarks are generated by a similar lagrangian, again without the extra quarks.

3. Charged currents

The presence of the extra $\tilde{t}$ quark modifies the electroweak currents. Now that the number of up-type quarks is four the matrix relating the quark mass eigenstates with the weak eigenstates becomes a 4×3 matrix in the LH model. The standard model quark doublets couple only to the gauge group $\mathrm{SU}(2)_1$. The charged currents in the LH model are given by [2]

$$\begin{aligned} \mathcal{L}_{CC} &= \frac{g_1}{\sqrt{2}} W_1^{\mu+} \bar{U}_L \gamma_\mu D_L + \text{h.c.} \\ &= \frac{g}{\sqrt{2}} W^{\mu+} V_{ij} \bar{U}_{iL}^m \gamma_\mu D_{jL}^m - \frac{g}{\sqrt{2}\tan\theta} W_H^{\mu+} V_{ij} \bar{U}_{iL}^m \gamma_\mu D_{jL}^m + \mathcal{O}\left(\frac{v^2}{f^2}\right) + \text{h.c.} \end{aligned} \tag{3.1}$$

Here V_{ij} is elements of the "extended" Cabibbo Kobayashi Maskawa (CKM) matrix where the subindices $i(j)$ run over 1 to 4(3), the number of generations in the up(down) sector. D_L and D_L^m are left-handed quarks in the down sector in weak and mass eigenstates respectivley. They are related by $U_D^m = T_L^D D_L$ with $D_L = (d_1, s_2, b_3)^T$ and $D_L^m = (d_L, s_L, b_L)^T$. The electroweak coupling constant is defined as $g = g_1 \sin\theta$. In eq. (18) we ignore small corrections arising from the Higgs vev. We are not concerned with it in this case because the corrections do not change the extended CKM matrix. Note that there are new charged currents associated with heavy gauge bosons W_H, and whose coupling constant is $g/\tan\theta$. A heavy copy of the SM gauge bosons causes interference effects, which are suppressed by $M_W^2/M_{W_H}^2$ compared with processes mediated only by the weak gauge boson W. For simplicity, we ignore the charged currents associated with the heavy gauge bosons W_H.

Now we find the expression for the extended CKM matrix $V = (T_L^U)^\dagger T_L^D$. It is convenient to work in a basis where the down-type quark mass eigenstates are identified with the weak eigenstates by setting $T_L^D = \mathbb{1}$ (unit matrix). The extended CKM matrix is then

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expressed by the elements of the up-type quark transformation matrix T_L^U only:

$$V_{ij}^{LH} = (T_L^U)_{ij} \qquad \text{for} \qquad i = 1,2,3,4\,; \qquad j = 1,2,3\,. \tag{3.2}$$

From now on we express the up-type quark transformation matrix T_L^U as follows:

$$T_L^U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & \Theta_u \\ V_{cd} & V_{cs} & V_{cb} & \Theta_c \\ V_{td} & V_{ts} & V_{tb} & \Theta_t \\ V_{Td} & V_{Ts} & V_{Tb} & \Theta_T \end{pmatrix}. \tag{3.3}$$

Compared to the CKM matrix in the standard model the extended CKM matrix has the fourth row elements V_{Td}, V_{Ts} and V_{Tb}. These parameters can be measured by the decays of mesons composed of the T quark and the down-type quarks. The inclusive decay rate $T \to q\ell\bar{\nu}$ is given by

$$\Gamma(T \to X_q \ell\bar{\nu}) \approx \frac{G_F^2 |V_{Tq}|^2}{192\pi^3} m_T^5 \approx 23 \times |V_{Tq}|^2 \left[\frac{m_T}{1\,\text{TeV}}\right]^5 \text{GeV}\,, \tag{3.4}$$

where q is down-type and m_T is the T quark pole mass. For the b quark the value of V_{Tb} is estimated in ref. [2].

$$|V_{Tb}| \sim \frac{|\lambda_{33}|^2}{|\lambda_{33}|^2 + |\lambda_0|^2} \frac{v}{f}\,. \tag{3.5}$$

For $\lambda_{33} \sim \lambda_0 \sim 1$ and $f \sim 1\,\text{TeV}$, the decay rate $T \to b\ell\bar{\nu}$ is given by

$$\Gamma(T \to b\ell\bar{\nu}) \sim 0.023 \times \left[\frac{m_T}{1\,\text{TeV}}\right]^5 \text{GeV}\,. \tag{3.6}$$

The dominant partial decay widths of the T quark are given in ref. [2]

$$\Gamma(T \to th) = \Gamma(T \to tZ) = \frac{1}{2}\Gamma(T \to bW^+) = \frac{1}{32\pi} \frac{|\lambda_{33}|^4}{|\lambda_{33}|^2 + |\lambda_0|^2} m_T\,. \tag{3.7}$$

Other decay modes are effectively suppressed by v^2/f^2. The branching ratio of the inclusive decay $T \to b\ell\nu$ is given by

$$\text{Br}(T \to b\ell\nu) \sim 1.2 \times 10^{-3} \times \left[\frac{m_T}{1\,\text{TeV}}\right]^4\,. \tag{3.8}$$

4. Neutral currents

In the standard model the neutral-current interactions do not change the flavor at the tree level. The smallness of the flavor-changing neutral currents(FCNC) transitons in the standard model is due to quantum loop effect and the GIM cancellation mechanism. In contrast, the FCNC in the LH model occurs at the tree level by the lagrangian [2]

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} Z_\mu \left(J_{W^3}^\mu - \sin^2\theta_W J_{EM}^\mu\right) + \frac{g}{\tan\theta} Z_{H\mu} J_{W^3}^\mu + \mathcal{O}\left(\frac{v^2}{f^2}\right), \tag{4.1}$$

where J^μ_{EM} is the electric current which is the same as in the standard model, and the weak neutral current $J^\mu_{W^3}$ is given by

$$\begin{aligned} J^\mu_{W^3} &= \frac{1}{2}\bar{U}_L\gamma^\mu U_L - \frac{1}{2}\bar{D}_L\gamma^\mu D_L \\ &= \frac{1}{2}\bar{U}^m_L\gamma^\mu\,\Omega\,U^m_L - \frac{1}{2}\bar{D}^m_L\gamma^\mu D^m_L\,. \end{aligned} \tag{4.2}$$

Note that the neutral currents in the down sector remains the same as those in the standard model while those in the up sector have additional currents associated with the T quark. The up-type quark transformation matrix generates a 4×4 neutral currents mixing matrix Ω in the up sector. The elements of Ω is then expressed only by the fourth column elements of T^U_L:

$$\begin{aligned} \Omega &= T^U_L\,\mathrm{diag}(1,1,1,0)\,T^{U\dagger}_L \\ &= \begin{pmatrix} 1-|\Theta_u|^2 & -\Theta_u\Theta^*_c & -\Theta_u\Theta^*_t & -\Theta_u\Theta^*_T \\ -\Theta_c\Theta^*_u & 1-|\Theta_c|^2 & -\Theta_c\Theta^*_t & -\Theta_c\Theta^*_T \\ -\Theta_t\Theta^*_u & -\Theta_t\Theta^*_c & 1-|\Theta_t|^2 & -\Theta_t\Theta^*_T \\ -\Theta_T\Theta^*_u & -\Theta_T\Theta^*_c & -\Theta_T\Theta^*_t & 1-|\Theta_T|^2 \end{pmatrix}. \end{aligned} \tag{4.3}$$

Note that the diagonal elements are less than 1. Furthermore, it holds that $1-|\Theta_i|^2 = |V_{id}|^2+|V_{is}|^2+|V_{ib}|^2$ for $i=u,c,t,T$ due to unitarity of the matrix T^U_L. From the Particle Data Book we have the upper bound on $|\Theta_i|$: $|\Theta_u|<0.091$, $|\Theta_c|<0.147$, $|\Theta_t|<0.997$.

Unitarity of the up-type transformation matrix T^U_L also requires that the off-diagonal elements do not vanish. The standard model unitarity triangle should be replaced by a unitary quadrangle in the LH model. For eaxmple, unitarity applied to the first and the second column yields

$$V_{ud}V^*_{cd}+V_{us}V^*_{cs}+V_{ub}V^*_{cb}+\Theta_u\Theta^*_c=0\,. \tag{4.4}$$

Then the last term on the left side becomes Ω_{uc}.

$$\Omega_{uc}=-\Theta_u\Theta^*_c=V_{ud}V^*_{cd}+V_{us}V^*_{cs}+V_{ub}V^*_{cb}\neq 0\,. \tag{4.5}$$

There is a way of estimating the absolute value of Θ_i, $(i=u,c,t,T)$ from up-type quark mass matrix $\mathcal{M}_U$. The details of the calculation are given in the appendix. Note that the values of $|\Theta_i|$ are related to the third column of the up-type quark transformation matrix T^U_L.

$$|\Theta_u| \sim |V_{ub}|\frac{v}{f} \sim 0.001 \tag{4.6}$$

$$|\Theta_c| \sim |V_{cb}|\frac{v}{f} \sim 0.01 \tag{4.7}$$

$$|\Theta_t| \sim |V_{tb}|\frac{v}{f} \sim 0.25 \tag{4.8}$$

$$|\Theta_T| \sim 0.93\,. \tag{4.9}$$

The last relation comes from the unitarity of the matrix T^U_L: $|\Theta_u|^2+|\Theta_c|^2+|\Theta_t|^2+|\Theta_T|^2=1$. The numerical values are estimated for $f\sim 1\,\mathrm{TeV}$. The off-diagonal elements of the matrix

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Ω are then given by

$$|\Omega_{uc}| = |\Theta_u||\Theta_c| \sim 10^{-5} \tag{4.10}$$
$$|\Omega_{tu}| = |\Theta_t||\Theta_u| \sim 2.5 \times 10^{-4} \tag{4.11}$$
$$|\Omega_{tc}| = |\Theta_t||\Theta_c| \sim 2.5 \times 10^{-3}\,. \tag{4.12}$$

An interesting feature is that the up-type quark anomalous couplings allow the Z-mediated FCNC at tree level only in the up sector. This may modify the SM predictions for rare D meson decays, $D^0 - \bar{D}^0$ mixing or $c \to u$ penguin processes. This can also provide the possibility of rare top quark decays and same sign top pair production at the LHC as a direct probe of the Z-mediated FCNC. In what follows, we consider the processes in the framework of the LH model.

4.1 Rare D meson decays

Hadronic D meson decays are completely dominated by nonperturbative physics and in general do not constitute a suitable test of the short distance structure of the standard model. But leptonic modes such as $D^0 \to l^+l^-$ can be used to constrain the size of the Z-mediated FCNC couplings in any extension of the standard model. In the LH model the SM gauge boson Z coupling $\Omega_{uc}Z\bar{u}_Lc_L$ can give rise to the decays $D^0 \to e^+e^-$, $\mu^+\mu^-$ at tree level. The contribution of the Z-mediated tree level diagram to the branching ratio of $D^0 \to \mu^+\mu^-$, normalized to that of the W-mediated $D^+ \to \mu^+\nu$, is given by

$$\frac{\mathrm{Br}(D^0 \to \mu^+\mu^-)_Z}{\mathrm{Br}(D^+ \to \mu^+\nu)} = 2\left[\frac{\tau(D^0)}{\tau(D^+)}\right][(1/2 - \sin^2\theta_W)^2 + \sin^4\theta_W]\frac{|\Omega_{uc}|^2}{|V_{cd}|^2}\,. \tag{4.13}$$

Using eq. (4.10) we estimate the ratio:

$$\frac{\mathrm{Br}(D^0 \to \mu^+\mu^-)_Z}{\mathrm{Br}(D^+ \to \mu^+\nu)} \sim 2 \times 10^{-10}\,. \tag{4.14}$$

Using the experimental bound $\mathrm{Br}(D^+ \to \mu^+\nu) = (3.5 \pm 1.4) \times 10^{-4}$ [8] the branching ratio of $D^0 \to \mu^+\mu^-$ is given by

$$\mathrm{Br}(D^0 \to \mu^+\mu^-) \sim 7 \times 10^{-14}\,. \tag{4.15}$$

The estimated SM branching ratio is $\sim 10^{-19}$ [10], and the present experimental bound is 1.3×10^{-6} [9], still undetectable by foreseeable experments.

4.2 $D^0 - \bar{D}^0$ mixing

In this section we discuss how the LH model affects $D^0 - \bar{D}^0$ mixing. The recent experiments confirm that $D^0 - \bar{D}^0$ mixing proceeds very slowly [11]. The Stanard model explains that short distance contributions to $D^0 - \bar{D}^0$ mixing occurs via box diagrams, and the s-quark contribution in the box diagrams is dominant. The b-quark contribution is much smaller due to the small CKM elements $|V_{ub}V_{cb}^*|^2/|V_{us}V_{cs}^*|^2 = \mathcal{O}(10^{-6})$, and due to the relative smallness of m_b [12, 13]. That is, the Glashow-Ilipoulos-Maiani (GIM) cancellation is

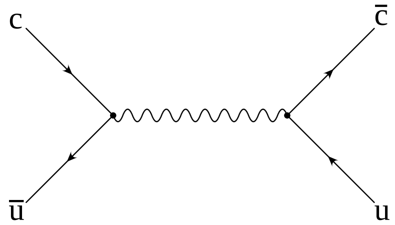

Figure 1: Tree-level FCNC $D^0 - \bar{D}^0$ mixing diagram contributes only to the x parameter.

very effective in the $D^0 - \bar{D}^0$ system compared with that in the $B^0 - \bar{B}^0$ system. In contrast, long distance contributions to $D^0 - \bar{D}^0$ mixing are inherently nonperturbative, and cannot be calculated from the first principles. Interestingly, mixing from the long distance effects can dominate over the short distance ones, and even reach the current experimental limit [14, 15].

We now review the current experimental data for charm mixing. Two physical parameters that characterize the mixing are

$$x = \frac{\Delta m}{\Gamma}, \qquad y = \frac{\Delta \Gamma}{2\Gamma}, \tag{4.16}$$

where Δm and $\Delta\Gamma$ are mass and width differences of the two neutral D meson mass eigenstates and Γ is their averaged width. The present upper bounds on x and y are at a few times 10^{-2} level [11]. In ref. [14], it is asserted that the values of $|x|$ and $|y|$ are between 10^{-3} and 10^{-2} within the standard model. To check theoretical prediction more precisely, one need to improve sensitivity at least at 0.1% level in the future generation of charm collider experiments.[1] In some cases, even the current experimental limit is enough to severely constrain new physics beyond the standard model.

In the LH model, the anomalous coupling Ω_{uc} produces $D^0 - \bar{D}^0$ mixing at tree level by the Z boson exchange, as depicted in figure 1, and results in the short distance $\Delta C = 2$ transition contributing only to x. In what follows, we compute only the LH model contribution. The mass difference due to Z boson Δm_D^Z is given by

$$(\Delta m_D)^Z = \frac{\sqrt{2}}{3} G_F m_D f_D^2 B_D \eta_D |\Omega_{uc}|^2, \tag{4.17}$$

where m_D is mass of the D^0 meson, f_D is decay constant of the D^0 meson, a QCD correction η_D is ~ 0.8, and $B_D = 1$ in vaccum saturation approximation. These give the expression

$$(\Delta m_D)^Z \approx 2 \times 10^{-7} \times |\Omega_{uc}|^2 \,\text{GeV}. \tag{4.18}$$

For $|\Omega_{uc}| \sim 10^{-5}$, the mass difference is estimated to be $\sim 10^{-17}$ GeV, and this corresponds to $x \sim 10^{-5}$. One notes that the standard model prediction for charm mixing is much larger than the LH model prediction by at least a factor of 100. As for y, one expects that the Z mediated process contributes only to $\Delta C = 2$ and the LH model does not give any significant contribution to y.

[1] Due to uncertainty from the standard model prediction for charm mixing, the only robust potential signal of new physics in charm system at this stage comes from large CP violation.

4.3 $t \to cZ$ decay

The LHC will produce millions of top quarks at the detectors and most top quarks decay to bW^+ by weak interactions. There can also be rare top quark decays like $t \to cZ$ with a real Z boson. In the SM it occurs at one-loop level, and its branching ratio is predicted to be $\sim 10^{-12}$ [16]. In contrast, in the LH model $t \to cZ$ decay occurs at tree level and the decay width is given by

$$\begin{aligned} \Gamma(t \to cZ) &= |\Omega_{tc}|^2 \frac{g^2}{128\pi \cos^2\theta_W} \frac{m_t^3}{m_Z^2} \left(1 - \frac{m_Z^2}{m_t^2}\right)^2 \left(1 + 2\frac{m_Z^2}{m_t^2}\right) \\ &\approx 1.0 \times |\Omega_{tc}|^2 \,\text{GeV}\,. \end{aligned} \tag{4.19}$$

Using eq. (4.12) the decay width is estimated by

$$\Gamma(t \to cZ) \sim 6 \times 10^{-6}\,\text{GeV}\,. \tag{4.20}$$

Assuming that the total width of the top quark is dominated by $t \to bW^+$, the branching ratio of $t \to cZ$ decay in the LH model is given by

$$\text{Br}(t \to cZ) \approx \frac{\Gamma(t \to cZ)}{\Gamma(t \to bW^+)} \approx \frac{1}{2\cos^2\theta_W} \frac{|\Omega_{tc}|^2}{|V_{tb}|^2} \frac{m_W^2}{m_Z^2} \approx 3 \times 10^{-6}. \tag{4.21}$$

By ignoring the up and charm quark masses and replacing the anomalous couplings, one can compute the branching ratio of $t \to uZ$ decay.

$$\text{Br}(t \to uZ) \sim 5 \times 10^{-11}\,. \tag{4.22}$$

The sensitivity of ATLAS experiment to the FCNC decay $t \to Zq$ (with $q = u, c$) has been analyzed [17] by searching for a signal in the channel $t\bar{t} \to (Wb)(Zq)$, with the boson being reconstructed via the leptonic decay $Z \to \ell\ell$. The $t \to Z(u,c)$ signal in the ATLAS should be very clean but, due to the low signal event rare and large backgrounds, only $\sim 3 \times 100\text{fb}^{-1}$ of integrated luminosity would allow one to probe $\text{Br}(t \to Z(u,c))$ as low as 10^{-4} [18]. Therefore, the LH model prediction for decay $t \to Zq$ can not be tested in the ALTAS and CMS detectors.

4.4 Like-sign tt ($\bar{t}\bar{t}$) pair production

The LHC will begin to outline the physics at TeV energy scale. With an integrated luminosity of about 100 fb^{-1} the LHC is expected to produce several tens of millions of $t\bar{t}$ pairs for each of the detectors, ATLAS and CMS per year [18]–[20]. Such a high rate of production will allow the LHC to search for new physics associated with the top (anti-top) quark. For example, one take into account a process in which two incoming up-type quarks exchange a neutral gauge bosons like gluon, γ, Z and then change to a pair of (same sign) top quarks[21], as depicted in figure 2.

In this section, we consider the production cross section for $pp \to tt$ or $\bar{t}\bar{t}$ processes in the framework of the LH model. The dimension 4 operators with the anomalous couplings Ω_{tq} ($q = u, c$) produce a signal of same sign top quark pairs at tree level by the exchange

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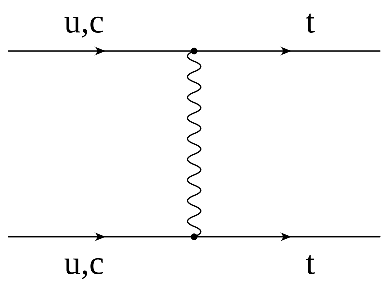

Figure 2: t-channel for same sign tt production process (There is also u-channel diagram.)

of Z.

$$\frac{g}{2\cos\theta_W} Z_\mu (\Omega_{tu}\, \bar{t}_L \gamma^\mu u_L + \Omega_{tc}\, \bar{t}_L \gamma^\mu c_L) + \text{h.c.} \tag{4.23}$$

To compute the production cross section for $pp \to tt$ process, one takes into account $uu, uc, cc \to tt$ processes at the parton level. Due to the larger parton luminosity from the u quark one expect that the process $uu \to tt$ dominates over the process $cc \to tt$. However, one note that the top-charm anomalous coupling is larger than the top-up anomalous coupling by a factor of 10 in the LH model. Thus we consider the c quark as well as the u quark. Then the differential cross section for $pp \to tt$ process is given in parton variables as follows

$$\begin{aligned} \frac{d^3\sigma}{dx_1 dx_2 d\hat{t}}(pp \to tt + X) \;=\; & f_u(x_1) f_u(x_2) \frac{d\sigma}{d\hat{t}}(uu \to tt) + \\ & + 2 f_u(x_1) f_c(x_2) \frac{d\sigma}{d\hat{t}}(uc \to tt) + \\ & + f_c(x_1) f_c(x_2) \frac{d\sigma}{d\hat{t}}(cc \to tt)\,, \end{aligned} \tag{4.24}$$

where x_i is the longitudinal fraction of the proton's momentum, $\hat{t}$ is the conventional Mandelstam variable, and $f_q(x)$ is the parton distribution function (pdf) of q quark in the proton. There are the equivalent $\bar{u}\bar{u}, \bar{u}\bar{c}, \bar{c}\bar{c} \to \bar{t}\bar{t}$ like-sign anti-top processes. The differential cross section for these processes is given by

$$\begin{aligned} \frac{d^3\sigma}{dx_1 dx_2 d\hat{t}}(pp \to \bar{t}\bar{t} + X) \;=\; & f_{\bar{u}}(x_1) f_{\bar{u}}(x_2) \frac{d\sigma}{d\hat{t}}(\bar{u}\bar{u} \to \bar{t}\bar{t}) + \\ & + 2 f_{\bar{u}}(x_1) f_{\bar{c}}(x_2) \frac{d\sigma}{d\hat{t}}(\bar{u}\bar{c} \to \bar{t}\bar{t}) + \\ & + f_{\bar{c}}(x_1) f_{\bar{c}}(x_2) \frac{d\sigma}{d\hat{t}}(\bar{c}\bar{c} \to \bar{t}\bar{t})\,. \end{aligned} \tag{4.25}$$

Here we do not directly compute the production cross sections. Instead, we quote the result of the computation in ref. [22] where the anomalous coupling a^L_{tq} are defined as $a^L_{tq} \equiv \frac{g}{2\cos\theta_W}\Omega_{tq}$, and the plot in figure 2 shows the LHC production cross section in parton levels. For $|\Omega_{tu}| \sim 2.5 \times 10^{-4}$ and $|\Omega_{tc}| \sim 2.5 \times 10^{-3}$, the production cross section for $pp \to tt$ process is less than 1 event for a 100 fb^{-1} data sample. This rate is lower than the sensitivity of the LHC to the same sign $tt(\bar{t}\bar{t})$ pair production, and it is due to small top quark anomalous couplings in the LH model.

Note that converting a cross section measurement into a measurement of the coupling constant requires knowledge of the pdfs in the proton. The pdfs of the c and $\bar{c}$ quarks in the proton coincide while those of u and $\bar{u}$ quarks in the proton do not. It causes the difference of production cross sections for the $pp \to tt$ process and for the $pp \to \bar{t}\bar{t}$ process. These lead to direct measurement of the coupling constants $|\Omega_{tu}|$ and $|\Omega_{tc}|$. The numerical computation of the coupling constants is found in ref. [21].

5. Conclusion

The littlest Higgs model attributes the lightness of the Higgs to being a pseudo Goldstone boson, and provides a simple mechanism for electroweak symmetry breaking by introducing a heavy vector-like quark to trigger a negative mass squared parameter for the Higgs doublet. The presence of the new vector-like quark, as a consequence, modifies the standard model predictions for the flavor neutral changing currents as well as for the weak charged currents. The extended 4×3 CKM matrix contains extra three mixing parameters, while the neutral current mixing matrix consists of four new complex parameters. These parameters induce a couple of rare processes for the up-type quarks. We have showed that the LH model predictions for the mixing angles are too low compared to the current experimental bounds, and the testability of the model requires more stringent measurements of the mixing angles at the ATLAS and CMS detectors of the LHC in the future. We conclude that the quark sector of the LH medel is not a suitable test of the signals of the LH medel. There are more complex little Higgs models which demand a heavy vector-like quark just as the LH model does. As long as the models have only a vector-like quark they might show the similar characteristic signatures in the flavor neutral changing currents as well as in the charged currents. To distinguish other Little Higgs models from the littlest Higgs model, one should search for new signals in the gauge sector and in the Higgs sector as well.

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A. Elements of the unitary matrix T_L^U

The matrix T_L^U can be computed by diagonalizing the hermitean matrix $\mathcal{M}_U \mathcal{M}_U^\dagger$:

$$\mathcal{M}^2_{\rm diag} = T_L^U \, \mathcal{M}_U \mathcal{M}_U^\dagger \, T_L^{U\dagger} \tag{A.1}$$

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where $\mathcal{M}_{\text{diag}}$ is given by

$$\mathcal{M}^2_{\text{diag}} = \begin{pmatrix} m_u^2 & 0 & 0 & 0 \\ 0 & m_c^2 & 0 & 0 \\ 0 & 0 & m_t^2 & 0 \\ 0 & 0 & 0 & m_T^2 \end{pmatrix} \tag{A.2}$$

with m_q is the q quark mass. The up-type quark mass matrix $\mathcal{M}_U$ is given by eq. (17). The mass matrix squared $\mathcal{M}_U \mathcal{M}_U^\dagger$ is then expressed as follows:

$$\mathcal{M}^2_{U,\text{diag}} = T_{U,L}\, \mathcal{M}_U \mathcal{M}_U^\dagger\, T_{U,L}^\dagger \tag{A.3}$$

$$\mathcal{M}_U \mathcal{M}_U^\dagger = f^2 \left(\begin{array}{cccc} & & & -i\lambda_{13}\lambda_{33}^* \frac{v}{f} \\ & \mathbf{\Lambda}\mathbf{\Lambda}^\dagger (\frac{v}{f})^2 & & -i\lambda_{23}\lambda_{33}^* \frac{v}{f} \\ & & & -i|\lambda_{33}|^2 \frac{v}{f} \\ i\lambda_{13}^*\lambda_{33}\frac{v}{f} & i\lambda_{23}^*\lambda_{33}\frac{v}{f} & i|\lambda_{33}|^2\frac{v}{f} & |\lambda_{33}|^2 + |\lambda_0|^2 \end{array} \right) \tag{A.4}$$

where $(\Lambda)_{ij} = \lambda_{ij}$. The matrix equation is so complicated that the perturbation method is a good way to solve it. Then the problem is to solve eigenvalue problem in quantum mechanics with four eigenstates.

Set $\frac{v}{f} \leq \frac{1}{4}$ by choosing $f \geq 1\,\text{TeV}$.

$$|\lambda_0| \sim |\lambda_{33}| \gg |\lambda_{ij}| \qquad \text{where} \quad (i,j) \neq (3,3) \tag{A.5}$$

Each state is a mass eigenstate for the up-type quark and the coefficient of the states is the elements of $T_{U,L}$. The unperturbed hamiltonian is then given by

$$H_0 = f^2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & |\lambda_{33}|^2 (\frac{v}{f})^2 & -i|\lambda_{33}|^2 \frac{v}{f} \\ 0 & 0 & i|\lambda_{33}|^2 \frac{v}{f} & |\lambda_{33}|^2 + |\lambda_0|^2 \end{pmatrix} \tag{A.6}$$

and other elements are the perturbed hamiltonian H'. Then the coefficients are given to leading order by

$$\Theta_u \approx \frac{\langle u|H'|T\rangle}{m_u^2 - m_T^2}\,, \tag{A.7}$$

where $|u\rangle$ and $|T\rangle$ are unpertured quark mass eigenkets, and m_u and m_T are masses of u and T quarks respectively. The value of $|\Theta_u|$ is then estimated as

$$|\Theta_u| \sim \frac{|\lambda_{13}\lambda_{33}^*|}{|\lambda_{33}|^2 + |\lambda_0|^2} \frac{v}{f}\,. \tag{A.8}$$

The value of $|V_{ub}|$ can be estimated in the same way:

$$V_{ub} \approx \frac{\langle u|H'|t\rangle}{m_u^2 - m_t^2} \tag{A.9}$$

$$|V_{ub}| \sim \frac{|\lambda_{13}\lambda_{33}^*|}{|\lambda_{33}|^2 + |\lambda_0|^2} \frac{v^2}{f^2}\,. \tag{A.10}$$

Therefore, the value of $|\Theta_u|$ is estimated from the $|V_{ub}|$.

$$|\Theta_u| \sim |V_{ub}|\frac{v}{f}\,. \tag{A.11}$$

Likewise,

$$|\Theta_c| \sim |V_{cb}|\frac{v}{f} \tag{A.12}$$

$$|\Theta_t| \sim |V_{tb}|\frac{v}{f}\,. \tag{A.13}$$

Then the anomalous coupling for the up and charm quarks is estimated as

$$|\Omega_{uc}| = |\Theta_u||\Theta_c| \sim |V_{ub}||V_{cb}|\frac{v^2}{f^2}\,. \tag{A.14}$$

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